

2.4. English Form Phrases: Translation Variations

A striking difference between English and the formal language is the size of their vocabularies. While the formal language is lean to the point of dullness, English offers many different words to express the same idea. This is good news for poets, but a challenge when we need to shoehorn such stylistic extravagance into the cramped confines of the formal language. For English offers not only a variety of names for a rose, but also for an “and” or “not”. These multiple English labels for the same logical concept are called **translation variations**. Before proceeding to thornier examples of logical form in English, we can quickly dispense with the minor complication posed by translation variations.

1. Negations. We know that English “not” is translated by the tilde, as in this example.

P: It is raining

It is **not** raining: $\sim P$

An obvious variation is the contracted form of “not”: “**n’t**”.¹ Another is the long-winded phrase “**it is not the case that**”.

P: It is raining

It is **not** raining: $\sim P$

It **isn’t** raining: $\sim P$

It is not the case that it is raining: $\sim P$

(Though there might seem no use for a phrase as wordy as “It is not the case that,” we’ll find it useful when we want to deny more complex sentences. What, for example, is the English language denial of “We’ll have both truffles and grog unless the store is closed”?)

¹ This includes uses of “n’t” in the informal phrase “**ain’t**”. If “P” stands for “It is broken,” then “It **ain’t** broken” translates as “ $\sim P$ ”.

The words “**fail to**” also indicate negations – as the sameness of meaning in the following sentences illustrates.²

Q: It rained yesterday

It **didn’t** rain yesterday: $\sim Q$

It **failed to** rain yesterday: $\sim Q$

We can also make denials using fragments of English smaller than a word – the **negation morphemes**.³ Examples include “**im-**,” “**in-**,” “**un-**,” and “**il-**”.

The argument is valid: R

The argument is **invalid**: $\sim R$

It is possible to beat Rex at mahjongg: S

It is **impossible** to beat Rex at mahjongg: $\sim S$

Barbie is able to free climb Bear’s Reach: T

Barbie is **unable** to free climb Bear’s Reach: $\sim T$

Surfing in a public pool is legal: U

Surfing in a public pool is **illegal**: $\sim U$

Since “not” is only one way of denying a sentence in English, it isn’t accurate to call all these examples “**‘not’ sentences**”. While we could call them “denials” (and sometimes will, informally), officially such a sentence is a **negation**. In formal language, “ $\sim P$ ” is the negation of “ P ”.

2. Conjunctions. Because “and” also has a number of translation variations in English, we likewise trade in the casual label “**‘and’ sentence**” for its technical counterpart “**conjunction**”. “It is sunny and it is cold” is an English conjunction, just as “ $(P \wedge Q)$ ” is a formal conjunction.

² A second, unrelated sense of “fail” is the opposite of “pass” – as in “Suki failed the quiz”. That sort of “fail” doesn’t count as a variation on “not”.

³ A **morpheme** is the smallest chunk of a language with its own meaning. (In the jargon of Linguistics, ordinary words such as “cat” and “go” are **free morphemes**; whereas morphemes that can’t stand on their own as complete words – such as “un-” and “im-” – are **bound morphemes**.)

An obvious variation on “and” is the two-word phrase “**both... and**”. Other variations are “**and also**,” “**and... too**,” “**and... as well**,” “**and yet**,” and “**yet**”.

P: It’s cold **Q:** It’s sunny

It’s cold **and** it’s sunny: $(P \wedge Q)$
 It’s **both** cold **and** sunny: $(P \wedge Q)$
 It’s cold, **and** it’s **also** sunny: $(P \wedge Q)$
 It’s cold, **and** it’s sunny **too**: $(P \wedge Q)$
 It’s cold, **and** it’s sunny **as well**: $(P \wedge Q)$
 It’s cold, **and yet** it’s sunny: $(P \wedge Q)$
 It’s cold, **yet** it’s sunny: $(P \wedge Q)$

The phrases “and yet” and “yet” carry an additional suggestion of incompatibility or opposition between the left and right parts of the conjunction. For example, saying “It’s cold **yet** it’s sunny” sounds alright, because sunny conditions tend to counteract cold ones. But “Horses are muscular, **yet** they’re powerful,” sounds strange, because being muscular isn’t opposed to being powerful. (By contrast, “Horses are muscular **and** they’re powerful” sounds fine, since “and” doesn’t convey a sense of opposition.)

All the same, “yet” and “and yet” assert both their left and right parts, just like “and”. So for logical purposes of truth and validity, the subtle difference in connotation isn’t typically a difference that makes a difference.

For the same reason we also count the phrases “**but**,” “**though**,” “**although**,” and “**even though**” as conjunction phrases, and translate them all as “ \wedge ”.

P: It’s sunny **Q:** It’s cold

It’s cold and it’s sunny: $(Q \wedge P)$	It’s cold though it’s sunny: $(Q \wedge P)$
It’s cold but it’s sunny: $(Q \wedge P)$	It’s cold, although it’s sunny: $(Q \wedge P)$
	It’s cold, even though it’s sunny: $(Q \wedge P)$

(Note that “but” can appear with the words “also” or “too”.

The movie is entertaining, **but also** informative.

Kids will enjoy the movie, **but** adults will like it **too**.

Since we earlier saw “also” and “too” accompanying “and,” here’s a further bit of linguistic evidence that “but” belongs in the same family as “and”.)

Finally, sentences with a **relative clause** can often be translated as conjunctions. A relative clause is a small sentence which is part of a larger sentence, and which describes something in the way an adjective does.

Lucretia is a **tall** engineering major.

Lucretia is a **quiet** engineering major.

Lucretia is an engineering major **who dresses in black**

The adjectives “tall” and “quiet” describe what kind of engineering major Lucretia is. But the relative clause “who dresses in black” describes in the same way: Lucretia is a dressing-in-black sort of engineering major. And a sentence with a relative clause makes the same double claim that a conjunction does.

Lucretia is an engineering major **and** Lucretia dresses in black

Lucretia is an engineering major **who dresses in black**

Each of these sentences asserts both of the following claims.⁴

Lucretia is an engineering major

Lucretia dresses in black

So we **treat a sentence with a relative clause as a conjunction in disguise**. As the following examples show, we can replace the word at the beginning of the relative clause (“who,” “which,” “that”) with “and,” and patch up the remainder to match the left half of the conjunction. (For instance, the

⁴ The fact that a conjunction asserts both its left and right parts is what made it innocent to break a conjunction up into two separate premises when putting an argument in standard form, back in 1.2 §4.

remainder of the relative clause “has a nice beach” is made a free-standing sentence by giving it the same subject – “San Diego” – as the left half of the conjunction.)

San Diego is a city [**that** has a nice beach]
San Diego is a city, **and** San Diego has a nice beach

The Cathedral of Learning is a building [**which** everyone should see].
The Cathedral of Learning is a building, and everyone should see
the Cathedral of Learning.

3. Disjunctions. We replace the informal label “**or** sentence” with the more technical term “**disjunction**”.

Just as “and” could appear with or without its left partner “both,” so “**either... or**” says the same as simple “**or**”. Less obviously, “**unless**” also serves as a translation variation of “or”.

P: Jake is home **Q:** Jake is at the Bel-Aire

Either Jake is home **or** he’s at the Bel-Aire (P ∨ Q)

Jake is home **or** he’s at the Bel-Aire (P ∨ Q)

Jake is home **unless** he’s at the Bel-Aire (P ∨ Q)

English usage allows these phrases to express two quite different sorts of claims. For instance, if the daily special in a restaurant includes soup **or** salad, and Kitty chooses that special expecting both soup **and** salad included, she’s misinterpreted the “or” intended. For that one low price the special offers soup **or** salad **but not both**. (Getting both costs extra.) This sort of disjunction – offering one option or the other, but not both – is an **exclusive “or”** (because it **excludes** the option of having both).

By contrast, if the cost of the special includes coffee and Kitty takes hers with both cream and sugar, she needn’t worry about added expense when asked if she wants “cream **or** sugar”. Here the price of the special includes cream, or sugar, **or both**. This is an **inclusive “or”**.

“Unless” is sometimes exclusive. So when we say

We’ll have a picnic **unless** it rains

the sentence is intended to rule out having both (that is: a picnic in the rain).

But the warning

You’ll fail Logic **unless** you study

doesn’t rule out the possibility of your both studying **and** failing the class – if, for example, you study hard but don’t show up for the exams. Since we don’t find an inclusive/exclusive ambiguity in negation or conjunction phrases, the fact that “unless” permits both readings is evidence that it’s a translation variation of “or”.

We follow logical tradition in giving the vel an **inclusive** reading. So

$(P \vee Q)$

means

P, or Q, or possibly both.

But we can still express an exclusive disjunction in the formal language. The wording yields the essential clues: since “both” always accompanies an “and,” the exclusive “or” sentence

P or Q but not **both**

is short for

P or Q but not **both P and Q**.⁵

⁵ We add “and” because it goes with “both”. We add “P” and “Q” as the left and right parts because we take this to be a case of **deleted repetition** – hence of having parts we’ve seen earlier in the sentence.

Recognizing “but” as a variation on “and,” we see how to translate the whole sentence.

$$\begin{array}{l} P \text{ or } Q \text{ but not both } P \text{ and } Q \\ ((P \vee Q) \wedge \sim (P \wedge Q)) \end{array}$$

Often we need to consider conversational context, and the meaning of the sentences being disjoined, to tell whether an inclusive or exclusive disjunction is intended. But our policy in this book keeps matters simple: we **translate English disjunctions as inclusive unless clearly indicated otherwise** (by the speaker saying “but not both,” or something equally unambiguous).

4. More Complex Form Phrases. So far we’ve translated each English form phrase with a single connective. But for some form phrases that doesn’t work.

“**Neither... nor**” is one example. It walks and talks like a form phrase, providing argument patterns that remain valid regardless of the subject matter. Here is one argument form.

$$\begin{array}{l} \text{Neither } \bullet \text{ nor } \blacktriangle . \\ \hline \therefore \text{Not } \bullet . \end{array}$$

And here are some intuitively valid English instances of this pattern.

$$\begin{array}{l} \text{We're having } \textbf{neither} \text{ truffles } \textbf{nor} \text{ grog.} \\ \hline \therefore \text{We're } \textbf{not} \text{ having truffles.} \end{array}$$

$$\begin{array}{l} \text{Logic is } \textbf{neither} \text{ difficult } \textbf{nor} \text{ boring.} \\ \hline \therefore \text{Logic is } \textbf{not} \text{ difficult.} \end{array}$$

But no single tilde, wedge, or vel will properly translate this phrase into the formal language.

We then face two options. We could introduce another connective into the formal language, as the formal counterpart to “neither... nor”. That would keep translation simple, at the cost of complicating the formal language.

A clue toward a second option comes in recognizing two familiar bits of form in “neither... nor”: “either... or” and the “n” of negation. And in fact a “neither... nor” has the same meaning as the **denial of an “either... or”**.

Q: Are we having either truffles or grog?

A1: No, we’re **not** having **either**.

A2: No, we’re having **neither** truffles nor grog.

A formal **negation of a disjunction** will thus translate “neither... nor”.

P: We’re having truffles

Q: We’re having grog.

We’re having **neither** truffles **nor** grog.

$$\sim(P \vee Q)$$

Our translation methods follow this second course: restricting the formal language to just three connectives, and translating “neither... nor” as a combination of two connectives.⁶

⁶ Though we will later explore the consequences of adding a symbol for “neither... nor” (along with various other connectives) to the formal language. See reading 3.9.

“Neither... Nor” and Conjunctions

It occurs rather naturally to English speakers that the following two sentences also seem equivalent in meaning.

“We’re having **neither** truffles **nor** grog.”

“We’re **not** having truffles **and** we’re **not** having grog.”

Using the same translation key as before, this second sentence is translated as follows.

$$(\sim P \wedge \sim Q)$$

And this suggests that our formal translation of “neither... nor” is equivalent to this conjunction, just as the above two English sentences are equivalent.

$$\begin{aligned} &\sim(P \vee Q) \\ &(\sim P \wedge \sim Q) \end{aligned}$$

As a matter of fact that’s true – as we’ll later show, using various formal methods.

Still, I resist translating a “neither... nor” sentence by the conjunction of negations, for a simple reason: the negation of a disjunction still mirrors the English grammar more faithfully. In “ $\sim(P \vee Q)$ ” we see both disjunction and negation, just as we earlier did in “neither... nor”.

So our **official translation of “neither... nor”** will be as the negation of a disjunction: “ $\sim(P \vee Q)$ ”

The phrase “**without**” likewise provides argument patterns valid regardless of subject matter.

● without ▲

∴ ●

● without ▲

∴ Not ▲

For instance, both of these English arguments are valid.

1. Trixie passed Chemistry without doing the labs.

∴ Trixie passed Chemistry.

1. Trixie passed Chemistry without doing the labs.

∴ Trixie didn’t do the labs.

We could add a new connective to translate “without”. But the same two-connective maneuver serves here. Note that the valid patterns we see in “without” also hold for a conjunction with a negated right part.

(● ∧ ~▲)

∴ ●

● without ▲

∴ ●

(● ∧ ~▲)

∴ ~▲

● without ▲

∴ ~▲

That suggests “P without Q” be translated as “ $(P \wedge \sim Q)$ ”. Further support comes from the intuitive sameness of meaning between a “without” sentence and a conjunction with negated right half.

Trixie passed Chemistry **without** doing the labs
 Trixie passed Chemistry **even though** she didn’t do the labs

Barbie passed me **without** saying “Hi”
 Barbie passed me **but** she didn’t say “Hi”

So we **translate “without”** as the two-connective cluster “ $(\wedge \sim)$ ”.

P: Trixie passed Chemistry **Q:** Trixie did the labs

Trixie passed Chemistry **without** doing the labs.⁷
 $(P \wedge \sim Q)$

Reconstructing the Right Side of a “Without” Sentence

Notice that we must reconstruct the right part of the “without” sentence to get a normal subject matter sentence in our translation key.

R: Barbie passed me
S: Barbie said “Hi”

Barbie passed me without *saying “Hi”*

“**Saying ‘Hi’**” is a compressed mini-sentence. To recover the full sentence “Barbie said ‘Hi’” for the translation key, we execute these three steps.

⁷ (Burgess 2009: XX) agrees with this translation of “without”.

First, since this collapsed sentence lacks a subject, give it the same subject appearing in the left subject matter sentence.

Barbie passed me
Barbie saying “Hi”

Second, clip the “-ing” from the verb.

Barbie say “Hi”

Third: have the verb discuss the same **time** that the left sentence does. Here the left sentence “Barbie passed me” discusses the past (as the “-ed” makes clear); so the right sentence does the same.

Barbie passed*ed* me
Barbie said “Hi”.

This ‘re-inflated’ sentence is suitable for a translation key.

R: Barbie passed me
S: **Barbie said “Hi”**

Summary: Chapter Two Form Phrases

Negation Phrases:

“Not P” $\sim P$

not
n’t
fail to

it is not the case that
Negation Morphemes
(un-, im-, in-, ...)

Conjunction Phrases:

“P and Q” $(P \wedge Q)$

and
both... and
and also
and... too
and yet

yet
but
though
although
even though

Sentences with relative clauses

Disjunction Phrases:

“P or Q” $(P \vee Q)$

or
either... or

unless

More Complex Phrases:

“Neither P nor Q” $\sim(P \vee Q)$

“P without Q” $(P \wedge \sim Q)$